

Universal zero-frequency Raman slope in a d -wave superconductor

W. C. Wu and J. P. Carbotte

*Department of Physics and Astronomy, McMaster University
Hamilton, Ontario, Canada L8S 4M1*

(February 1, 2008)

It is known that for an unconventional superconductor with nodes in the gap, the in-plane microwave or dc conductivity saturates at low temperatures to a universal value independent of the impurity concentration. We demonstrate that a similar feature can be accessed using channel-dependent Raman scattering. It is found that, for a $d_{x^2-y^2}$ -wave superconductor, the slope of low-temperature Raman intensity at zero frequency is universal in the A_{1g} and B_{2g} channels, but not in the B_{1g} channel. Moreover, as opposed to the microwave conductivity, universal Raman slopes are sensitive not only to the existence of a node, but also to different pairing states and should allow one to distinguish between such pairing states.

PACS numbers: 78.30.-j, 74.62.Dh, 74.25.Gz

The effect of impurity scattering has been of increasing interests in studies of High- T_c superconductors [1–6]. For superconductors with an order parameter which has nodes on the Fermi surface, it is well known that impurity scattering can lead to a finite density of quasiparticle states at zero energy (gapless excitations). In particular, in the strong resonant scattering limit, quasiparticle states can be strongly localized as a result of the short coherence length and mean free path [1] provided that the impurity concentration is low. This has some interesting observable consequences. Of equal interest is that, as first predicted by Lee [1], while the effective impurity scattering rate γ is quite different for different impurity concentrations and different scattering limits (for example, $\gamma \sim \Delta_0 e^{-\Delta_0 \tau}$ in the Born limit and $\gamma \sim \Delta_0 (\Delta_0 \tau)^{-\frac{1}{2}}$ in the unitary limit with $1/\tau$ the normal-state scattering rate and Δ_0 the maximum of the gap), the microwave conductivity saturates at low temperatures ($\sigma_0 = ne^2/\pi m \Delta_0$) and is independent of γ (or the impurity concentration). The experimental verifications of this universal feature gives unambiguous evidence that the order parameter in High- T_c materials exhibits nodes on the Fermi surface.

In the context of thermal conductivity, Graf *et al.* [6] have found a similar universality related to the microwave conductivity via the Wiedemann-Franz law. This has been confirmed in an experiment by Taillefer *et al.* [7] which measures the in-plane low-temperature thermal conductivity of $\text{YBa}_2\text{Cu}_3\text{O}_{6.9}$ at different Zn substitutions for Cu. In the present letter, we demonstrate how one can also study these universal features by doing channel-dependent Raman scattering experiments. It is found, in a $d_{x^2-y^2}$ -wave superconductor, that the low-temperature slope of the Raman intensity at zero frequency are universal in the A_{1g} and B_{2g} channels, but strongly dependent on the scattering rate ($\sim \gamma^2$) in the B_{1g} channel. As opposed to the microwave or thermal conductivity for which an entire Fermi surface average is taken and thus the universal feature is general for all the pairing states so long as gap nodes cross the Fermi surface (except for a scale factor), which channels saturate and are universal and which do not saturate in Raman scattering change with the gap symmetry. This channel-dependent universality exhibited in the Raman scattering thus gives one more method to test the symmetry of the pairing state in the High- T_c superconductors and is more powerful than microwave or thermal conductivity because of the additional selectivity involved in the Raman geometry which allows some information on the position of the nodes in the Brillouin zone to be obtained. This phenomenon should be of particular interest in the heavy fermion superconductors where the pairing states are considered to be more diverse.

The saturation of the low-temperature microwave conductivity in a d -wave superconductor, is a result of a cancellation between the value of the impurity-induced density of states at zero energy and the quasiparticle relaxation lifetime and arises only if there exists nodes in the order parameter on the Fermi surface. The saturation in the slope of the low-temperature zero-frequency Raman intensity can be understood in a similar manner, with an additional channel dependent feature unique to Raman which arises from the different dependence on the chosen Raman geometry vertices which pick up different contributions around the Fermi surface. In a $d_{x^2-y^2}$ -wave superconductor, the B_{2g} -channel Raman vertex selects preferentially states along the diagonals of the Brillouin zone (where the nodal lines are) and hence probes directly the low-lying quasiparticle excitations. Consequently the net result is similar to what is seen in the microwave conductivity case. In contrast in the B_{1g} channel, the Raman vertex has maximum weight along the k_x or k_y axes and zero weight along the diagonals, consequently one is effectively measuring a finite gap and no universality is observed.

The Raman intensity is proportional to the imaginary part of the zero-momentum limit generalized density response function

$$\chi_{\gamma\Gamma}(i\nu_n) = -T \sum_{\mathbf{k}, \omega_n} \text{Tr}[\hat{\gamma}(\mathbf{k})\hat{G}(\mathbf{k}, i\omega_n + i\nu_n)\hat{\Gamma}(\mathbf{k}, i\nu_n)\hat{G}(\mathbf{k}, i\omega_n)], \quad (1)$$

where Tr denotes a trace and $\hat{\gamma}(\mathbf{k}) = \hat{\tau}_3\gamma(\mathbf{k})$ is the bare Raman vertex with $\hat{\tau}_3$ the Pauli matrix and $\gamma(\mathbf{k}) = \frac{m}{\hbar^2}\mathbf{e}_s \cdot \frac{\partial^2 \xi_{\mathbf{k}}}{\partial k_x \partial k_i} \cdot \mathbf{e}_i$ (effective mass approximation [8]). Here $\xi_{\mathbf{k}}$ is the electronic dispersion relation of the superconducting layer and \mathbf{e}_i (\mathbf{e}_s) correspond to the polarizations of incident (scattered) photons. The renormalized Raman vertex $\hat{\Gamma}$ in (1) is given by

$$\hat{\Gamma}(\mathbf{k}, i\nu_n) = \hat{\tau}_3\gamma(\mathbf{k}) - \hat{\tau}_3 v_c T \sum_{\mathbf{k}', \omega_n} \text{Tr}[\hat{\tau}_3 \hat{G}(\mathbf{k}', i\omega_n + i\nu_n)\hat{\Gamma}(\mathbf{k}', i\nu_n)\hat{G}(\mathbf{k}', i\omega_n)], \quad (2)$$

where v_c is the Coulomb interaction. In Eq. (2), we have ignored the contribution to the vertex corrections due to the impurity potentials and two-particle pairing interactions and have included only the effect of the Coulomb interaction. For isotropic impurity scattering, it is sufficient to use the bubble diagram at small \mathbf{q} , while the inclusion of the pairing interaction vertex correction is shown to have little effect on the Raman spectra [9] and is particularly negligible at the low frequencies of interest. However, the effect of impurities is fully included in the single-particle Green's function \hat{G} in Eqs. (1) and (2). Substituting Eq. (2) into (1), one obtains

$$\chi_{\gamma\Gamma}(i\nu_n) = \chi_{\gamma\gamma}(i\nu_n) - \frac{\chi_{\gamma 1}(i\nu_n)\chi_{1\gamma}(i\nu_n)}{\chi_{11}(i\nu_n) - v_c^{-1}}, \quad (3)$$

where $\chi_{\gamma 1}(i\nu_n)$ is defined in the same way as $\chi_{\gamma\Gamma}(i\nu_n)$ with $\hat{\Gamma}$ replaced by $\hat{\tau}_3$ in (1), and so on.

In terms of the particle-hole space, the single-particle Green's function is given by $\hat{G}^{-1}(\mathbf{k}, i\omega_n) = i\tilde{\omega}_n\hat{\tau}_0 - \tilde{\xi}_{\mathbf{k}}\hat{\tau}_3 - \tilde{\Delta}_{\mathbf{k}}\hat{\tau}_1$, where $\tilde{\omega}_n$, $\tilde{\xi}_{\mathbf{k}}$, and $\tilde{\Delta}_{\mathbf{k}}$ are the impurity-renormalized Matsubara frequencies, electron energy spectrum, and gap. \hat{G} is related to the noninteracting Green's function $\hat{G}_0^{-1}(\mathbf{k}, i\omega_n) = i\omega_n\hat{\tau}_0 - \xi_{\mathbf{k}}\hat{\tau}_3 - \Delta_{\mathbf{k}}\hat{\tau}_1$ via the Dyson's equation $\hat{G}^{-1}(\mathbf{k}, i\omega_n) = \hat{G}_0^{-1}(\mathbf{k}, i\omega_n) - \hat{\Sigma}(\mathbf{k}, i\omega_n)$. We shall solve the self-energy $\hat{\Sigma}$ due to the impurity scattering. By expanding $\hat{\Sigma}(i\omega_n) \equiv \sum_{\alpha} \Sigma_{\alpha}(i\omega_n)\hat{\tau}_{\alpha}$ ($\alpha = 0, 1, 3$), one finds $i\tilde{\omega}_n = i\omega_n - \Sigma_0$, $\tilde{\xi}_{\mathbf{k}} = \xi_{\mathbf{k}} + \Sigma_3$, and $\tilde{\Delta}_{\mathbf{k}} = \Delta_{\mathbf{k}} + \Sigma_1$. Employing the usual T -matrix approximation, the self-energy is then given by $\hat{\Sigma}(\mathbf{k}, i\omega_n) = n_i \hat{T}(\mathbf{k}, \mathbf{k}, i\omega_n)$, where n_i is the impurity density and

$$\hat{T}(\mathbf{k}, \mathbf{k}', i\omega_n) = v_i(\mathbf{k}, \mathbf{k}')\hat{\tau}_3 + \sum_{\mathbf{k}''} v_i(\mathbf{k}, \mathbf{k}'')\hat{\tau}_3 \hat{G}(\mathbf{k}'', i\omega_n)\hat{T}(\mathbf{k}'', \mathbf{k}', i\omega_n). \quad (4)$$

Here $v_i(\mathbf{k}, \mathbf{k}') \equiv \langle \mathbf{k}' | v_i | \mathbf{k} \rangle$ is the impurity potential. If we consider only isotropic impurity scattering [$v_i(\mathbf{k}, \mathbf{k}') = v_i$], the T -matrix in (4) is left only with frequency dependence and can be solved to get $\hat{T}(i\omega_n) = [1 - v_i \hat{\tau}_3 \hat{G}(i\omega_n)]^{-1} v_i \hat{\tau}_3$ with the integrated Green's function $\hat{G}(i\omega_n) \equiv \sum_{\mathbf{k}} \hat{G}(\mathbf{k}, i\omega_n)$. One can expand $\hat{G}(i\omega_n) = \sum_{\alpha} G_{\alpha}(i\omega_n)\hat{\tau}_{\alpha}$ ($\alpha = 0, 1, 3$) with $G_{\alpha}(i\omega_n) \equiv 1/2 \sum_{\mathbf{k}} \text{Tr}[\hat{\tau}_{\alpha} \hat{G}(\mathbf{k}, i\omega_n)]$. For a superconductor with particle-hole symmetry and an odd-parity gap which is the case for the $d_{x^2-y^2}$ state, $G_1(i\omega_n) = G_3(i\omega_n) = 0$. This immediately gives

$$\Sigma_0 = \frac{n_i G_0}{c^2 - G_0^2}, \quad \Sigma_1 = 0, \quad \Sigma_3 = \frac{c n_i}{c^2 - G_0^2}, \quad (5)$$

where $c \equiv 1/v_i$. The result of $\Sigma_1 = 0$ is a reflection of the well-known result that in an unconventional superconductor with nodes in the gap and zero average, the gap remains unrenormalized due to the impurity scattering ($\tilde{\Delta}_{\mathbf{k}} = \Delta_{\mathbf{k}}$). Furthermore the effect of Σ_3 is absorbed into the chemical potential as usual and consequently $\tilde{\xi}_{\mathbf{k}} \equiv \xi_{\mathbf{k}}$. Equation (5) is convenient for discussing both the weak (Born) scattering ($c \gg 1$) and strong (resonant) scattering ($c \ll 1$) limit. In the normal state ($\Delta_{\mathbf{k}} = 0$), one can easily work out that $i\tilde{\omega}_n = i\omega_n + i(1/2\tau)$, where in the Born limit, the (isotropic) scattering rate $1/2\tau = 2\pi N(0)n_i v_i^2$, while in the resonant limit, $1/2\tau = n_i/2\pi N(0)$. Here $N(0) = m/2\pi\hbar^2$ is the density of states per spin on the Fermi surface.

One can use a general Raman vertex $\gamma_s(\mathbf{k}) = \gamma_s^0 + \gamma_s^1 f_s(\phi)$ to classify different symmetry channels [9,10] denoted by s . Here γ_s^0 represents the isotropic and γ_s^1 represents the anisotropic part of γ_s and ϕ is the azimuthal angle in the x - y plane. In the case of a cylindrical Fermi surface, $f_s(\phi) = \cos(2\phi)$ for the B_{1g} channel, $f_s(\phi) = \sin(2\phi)$ for the B_{2g} channel, and $f_s(\phi) = \cos(4\phi)$ for the A_{1g} channel. With the above Raman vertices and in the case of perfect screening ($v_c^{-1} \rightarrow 0$), Eq (3) is reduced to

$$\chi_{\gamma\Gamma}(i\nu_n) = (\gamma_s^1)^2 \left[\chi_s^2(i\nu_n) - \frac{[\chi_s^1(i\nu_n)]^2}{\chi_s^0(i\nu_n)} \right], \quad (6)$$

where we have defined

$$\chi_s^i(i\nu_n) = -T \sum_{\mathbf{k}, \omega_n} [f_s(\phi)]^i \text{Tr}[\hat{\tau}_3 \hat{G}(\mathbf{k}, i\omega_n + i\nu_n) \hat{\tau}_3 \hat{G}(\mathbf{k}, i\omega_n)]. \quad (7)$$

In (6), the isotropic term (γ_s^0) is dropped as it cancels due to Coulomb screening. In the following we shall limit ourselves to a $d_{x^2-y^2}$ -wave superconductor with gap $\Delta_{\mathbf{k}} = \Delta_0 \cos(2\phi)$. In both the cases of the B_{1g} and B_{2g} channels, the second term in (6) vanishes since $\chi_s^1(i\nu_n) = 0$. Technically this is due to the Fermi surface average $\langle f_s(\phi) |\Delta_{\mathbf{k}}|^2 \rangle = 0$. However this condition doesn't hold in the A_{1g} channel where the squared gap function has a component identical to $f_{A_{1g}}(\phi) = 4 \cos(4\phi)$ and hence $\langle \cos(4\phi) |\Delta_{\mathbf{k}}|^2 \rangle \neq 0$. As a consequence, the Coulomb screening only has an effect on the A_{1g} channel intensity and has no effect on the B_{1g} and B_{2g} channels.

Based on the cylindrical Fermi surface approach, the second term of Eq. (6) in the A_{1g} channel is shown to be of the same order as the first term in the zero-frequency limit [11]. In general, however, the effect of the second term is quite sensitive to the underlying quasiparticle energy dispersion relation and the issue regarding the effect of screening in the A_{1g} channel remains an issue of considerable debate [12]. On the other hand, experimental data seems to suggest that the first term of Eq. (6) can account well for the Raman intensity in the low-frequency regime. We thus drop the second term of (6) in our calculations of the slopes of the *channel dependent* Raman intensity at zero frequency in *all* channels. These are defined by ($i\nu_n \rightarrow \Omega + i\delta$)

$$S \equiv \left. \frac{d\chi_{\gamma\Gamma}''(\Omega)}{d\Omega} \right|_{\Omega \rightarrow 0} = \left. \frac{\chi_{\gamma\Gamma}''(\Omega)}{\Omega} \right|_{\Omega \rightarrow 0}, \quad (8)$$

where the double prime denotes an imaginary part and the equal sign arises because $\chi_{\gamma\Gamma}''(\Omega = 0) = 0$. Using the spectral representation for the imaginary frequency Green's function, analytically continuing to real frequency from imaginary frequency ($i\omega_n \rightarrow \omega + i\delta$), and then performing the frequency sum and momentum sum (replaced by an integration $\sum_{\mathbf{k}} = 2N(0) \int_{-\infty}^{\infty} d\xi \int_0^{2\pi} \frac{d\phi}{2\pi}$) gives

$$S = N(0) [\gamma_s]^2 \int_0^{2\pi} \frac{d\phi}{2\pi} [f_s(\phi)]^2 \int_{-\infty}^{\infty} d\omega \frac{f(\omega) - f(\omega - \Omega)}{\Omega} \\ \times \text{Im} \left[\frac{\tilde{\omega}'_+(\tilde{\omega}_+ + \tilde{\omega}'_+) - 2\Delta_{\mathbf{k}}^2}{(\xi_+ + \xi'_+)\xi_+\xi'_+} + \frac{\tilde{\omega}'_-(\tilde{\omega}_+ + \tilde{\omega}'_-) - 2\Delta_{\mathbf{k}}^2}{(\xi_+ - \xi'_-)\xi_+\xi'_-} \right]_{\Omega \rightarrow 0} \quad (9)$$

where $\tilde{\omega}_{\pm} \equiv i\tilde{\omega}_n(\omega \pm i\delta)$; $\tilde{\omega}'_{\pm} \equiv i\tilde{\omega}_n(\omega - \Omega \pm i\delta)$ and $\xi_{\pm} \equiv \text{sgn}(\omega) \sqrt{\tilde{\omega}_{\pm}^2 - \Delta_{\mathbf{k}}^2}$; $\xi'_{\pm} \equiv \text{sgn}(\omega - \Omega) \sqrt{(\tilde{\omega}'_{\pm})^2 - \Delta_{\mathbf{k}}^2}$ which are chosen to have branch cuts such that $\text{Im}\xi_+, \text{Im}\xi'_+ > 0$ and $\text{Im}\xi'_-, \text{Im}\xi_- < 0$. The index 1 in the vertex is dropped ($\gamma_s^1 \rightarrow \gamma_s$) for simplicity.

It is useful to compare Eq. (9) for the zero-frequency Raman slope with a similar expression for the microwave conductivity (see, for example, Eq. (2) of Ref. [2]). One finds that they are the same apart from an overall constant factor which appears in front of the expression and from a different angular function [in Raman scattering, the angular function $f_s(\phi) = \cos(4\phi)$, $\cos(2\phi)$, or $\sin(4\phi)$ for A_{1g} , B_{1g} , and B_{2g} channels, while in the conductivity the angular function is usually $\hat{p}_x = \cos(\phi)$ for calculating σ_0^{xx} or $\hat{p}_y = \sin(\phi)$ for calculating σ_0^{yy}]. Also the appearance of term $\Delta_{\mathbf{k}}^2$ in the second line of (9) is unique to Raman and occurs due to the different type of vertex (which is coupled to $\hat{\tau}_3$ in Raman and $\hat{\tau}_0$ in the conductivity). The term proportional to $\Delta_{\mathbf{k}}^2$, however, will drop out due to a cancellation between the two terms at zero temperature and contributes only a small amount at finite temperatures.

We consider first the $T = 0$ limit which gives $[f(\omega) - f(\omega - \Omega)]/\Omega \approx \partial f(\omega)/\partial \omega \approx -\delta(\omega)$ when $\Omega \rightarrow 0$. This means that the ω integration in (9) is sharply peaked around the small ω region centered at $\omega = 0$. Consequently we have

$$S = N(0) \gamma_s^2 \left\langle \frac{[f_s(\phi)]^2 \gamma^2}{(\gamma^2 + \Delta_{\mathbf{k}}^2)^{\frac{3}{2}}} \right\rangle, \quad (10)$$

where $\langle \cdots \rangle$ denotes an average over the Fermi surface. This expression agrees with one obtained before by Devereaux and Kampf [13]. Here the impurity-induced scattering rate in the superconducting state at zero-frequency is $\gamma = -i\tilde{\omega}(\omega = 0) = i\Sigma_0(\omega = 0)$. The self-consistent results for γ in the two different scattering limits Born and resonant

were solved for by Lee [1] as mentioned earlier. Assuming that $\gamma \ll \Delta_0$ (which requires that the impurity concentration n_i be small in the resonant limit), we find at $T = 0$

$$\begin{aligned} S &\sim \frac{m\gamma_s^2}{\pi^2\hbar^2\Delta_0} & s = B_{2g} \text{ or } A_{1g} \\ &\sim \frac{m\gamma_s^2}{\pi^2\hbar^2\Delta_0} \left(\frac{\gamma}{\Delta_0}\right)^2 \ln\left(\frac{\Delta_0}{\gamma}\right) & s = B_{1g}. \end{aligned} \quad (11)$$

As shown clearly in (11), the zero-frequency Raman slopes in both B_{2g} and A_{1g} -channel exhibit a universal saturated value at $T = 0$ which is independent of γ , *i.e.*, of impurity concentration – a feature first discovered by Lee [1] in the context of the microwave conductivity. The reason the B_{2g} and A_{1g} channels share the same limiting value is simply because the square of angular functions $[f_{B_{2g}}(\phi)]^2 = \sin^2(2\phi) = 1 - \cos^2(2\phi)$ and $[f_{A_{1g}}(\phi)]^2 = \cos^2(4\phi) = [1 - 2\cos^2(2\phi)]^2$ and the contribution due to whatever terms couple to $\cos(2\phi)$ is small. In contrast in the B_{1g} channel, the zero-frequency Raman slope is proportional to γ^2 (up to a logarithmic correction) and hence is strongly dependent on the impurity concentration. In the unitary limit, $\gamma \sim \tau^{-1/2} \sim n_i^{1/2}$, therefore the B_{1g} slope $S \sim n_i$. We recall that for a system with tight-binding bands, the Raman vertex strength $\gamma_{A_{1g}}$ and $\gamma_{B_{1g}}$ in (11) is proportional to the nearest-neighbor hopping, while $\gamma_{B_{2g}}$ is proportional to the next nearest-neighbor hopping [14].

If we take the ratio between the Raman intensities from superconducting and normal states, we find in the limits of $\Omega = 0$ and $T = 0$, $\chi''_S/\chi''_N \sim \gamma^2/\Delta_0^3\tau$ in the B_{1g} channel and $\sim 1/\Delta_0\tau$ in the B_{2g} channel. These differ from the expression given by Devereaux and Kampf [13] in that they use γ for $1/\tau$ in the normal state.

The finite but low-temperature ($T \lesssim \gamma$) limit is obtained by expanding $\tilde{\omega}_\pm = \pm i(\gamma + b\omega^2) + a\omega$ at small ω in (9), where γ , a and b are constants and are found to be $a \simeq 1/2$ and $b \simeq 1/(8\gamma)$ in the resonant scattering limit of primary interest here. Expanding the integrand in (9) to second order in ω leads to the finite- T result

$$\begin{aligned} S &\sim \frac{m\gamma_s^2}{\pi^2\hbar^2\Delta_0} \left(1 + \frac{\pi^2 T^2}{36 \gamma^2}\right) & s = B_{2g} \text{ or } A_{1g} \\ &\sim \frac{m\gamma_s^2}{\pi^2\hbar^2\Delta_0} \left(\frac{\gamma}{\Delta_0}\right)^2 \ln\left(\frac{\Delta_0}{\gamma}\right) \left(1 + \frac{\pi^2 T^2}{12 \gamma^2}\right) & s = B_{1g}. \end{aligned} \quad (12)$$

While the universality is channel dependent, the variation $\sim T^2$ is found in all three channels. Equivalent results were given by Hirschfeld *et al.* [2] for the microwave conductivity and by Graf *et al.* [6] for the thermal conductivity.

As mentioned before, the channel-dependent universal zero-frequency Raman slopes are sensitive to differences in pairing states. For completeness, we consider some other pairing states of interest and focus only on the B_{1g} and B_{2g} channels at $T = 0$. The opposite result to (11) with B_{1g} and B_{2g} channels switched, is obtained if the gap has d_{xy} symmetry with $\Delta_{\mathbf{k}} = \Delta_0 \sin(2\phi)$. In this case, one finds a universal feature in the B_{1g} -channel Raman slope, but not in the B_{2g} channel. This is because one sees mainly regions of maximum gaps in the B_{2g} -channel, while in the B_{1g} -channel, one sees mainly the regions of zero gap. For a system with extended s -wave pairing state, *i.e.*, with $\Delta_{\mathbf{k}} = \Delta_0 \cos(4\phi)$, we find that both B_{1g} and B_{2g} channels have the same universal Raman slope $S = m\gamma_s^2/2\pi^2\hbar^2\Delta_0$. In this case, the gap nodes appear at the angles of $\pi/8$, $3\pi/8$ and the equivalent and consequently both B_{1g} and B_{2g} channels see the same effective contribution from the gap node regions.

Finally, we consider the case of a mixing gap with a $d_{x^2-y^2}$ -wave component and a *small* isotropic s -wave component of weight α with the gap given by $\Delta_{\mathbf{k}} = \Delta_0[\cos(2\phi) + \alpha]$, where $\alpha < 1$. Physically this is somewhat similar to a system with orthorhombic band structure and a pure $d_{x^2-y^2}$ -wave gap. In contrast to the pure $d_{x^2-y^2}$ -wave case, the gap nodes are now shifted away from the diagonals and consequently the B_{1g} Raman channel will have some contribution from the node regions. This means that the universal zero-frequency Raman slope feature will also be present in the B_{1g} channel and its weight will depend on how far the gap node have shifted off the diagonals, *i.e.*, how much node contribution this channel picks up. To leading order, we find $S = m\gamma_s^2/\pi^2\hbar^2\Delta_0$ in the B_{2g} channel (same as the pure $d_{x^2-y^2}$ -wave case), while in the B_{1g} channel,

$$S_{B_{1g}} = \frac{m\gamma_s^2}{\pi^2\hbar^2\Delta_0} [\alpha^2 + \gamma^2/\Delta_0^2 \ln(\Delta_0/\gamma)]. \quad (13)$$

Therefore when $\alpha \gg \gamma/\Delta_0$, the B_{1g} Raman slope has universality which however breaks down when $\alpha \lesssim \gamma/\Delta_0$. In the unitary limit, $\gamma \sim n_i^{1/2}$ and thus $S_{B_{1g}} \sim m\gamma_s^2/\pi^2\hbar^2\Delta_0(\alpha^2 + cn_i)$. A study of the impurity concentration dependence

of $S_{B_{1g}}$ may be used to extract the size of α^2 which in turn enables one to test how orthorhombic the system is. We note that in the $d_{x^2-y^2}+s$ -wave case, $\langle \Delta_{\mathbf{k}} \rangle \neq 0$ and Σ_1 in Eq. (5) is nonzero and must be retained in the calculation. These complications are accounted for (see Ref. [11]) in obtaining the above result. Moreover, in this case, the second (screening) term of Eq. (6) contributes in the B_{1g} channel (but not in the B_{2g} channel), though this contribution is expected to be small when the s -wave component is small.

In conclusion, we have found that channel-dependent Raman scattering can be used to test a universal low-temperature behaviour in disorder high- T_c superconductors which in turn can reveal information on the pairing state in these materials. We studied the slopes of low-temperature Raman intensity at zero frequency in various channel. Similar to what was found in the microwave or dc conductivity [1] and thermal conductivity [6], for a $d_{x^2-y^2}$ -wave superconductor with nodal order parameter along the diagonals on the Fermi surface, the universal feature holds in both A_{1g} - and B_{2g} -channel Raman slopes. Such a feature is not found, however, in the B_{1g} -channel Raman slope. Moreover, in contrast to microwave or thermal conductivity where the universal features are general to all the pairing states with nodes and certain reflection symmetry, the channel-dependent universality or lack thereof in Raman scattering is sensitive to difference in pairing states and hence allows one to further clarify the symmetry of the pairing states. In addition for a mixed order parameter with $d_{x^2-y^2}$ and s symmetries, we have shown that Raman is able to give the amount of s -wave component which is not the case using microwave or thermal conductivity. Finally, we remark that, as recently shown by Branch [15], the strong inelastic scattering effect in the high- T_c materials may dominate over the elastic impurity scattering but will not destroy the universality in Raman slopes predicted here because this is a zero-temperature effect and at $T = 0$ the inelastic scattering time becomes infinite.

We are grateful to D. Branch for enlightening discussions. This work was supported by Natural Sciences and Engineering Research Council (NSERC) of Canada and Canadian Institute for Advanced Research (CIAR).

-
- [1] P. A. Lee, Phys. Rev. Lett. **71**, 1887 (1993).
 - [2] P. J. Hirschfeld, W. O. Putikka, and D. J. Scalapino, Phys. Rev. Lett. **71**, 3705 (1993); Phys. Rev. B **50**, 10250 (1994).
 - [3] P. Arberg, M. Mansor, and J. P. Carbotte, Solid State Comm. **86**, 671 (1993).
 - [4] P.J. Hirschfeld and N. Goldenfeld, Phys. Rev. B **48**, 4219 (1993).
 - [5] Y. Sun and K. Maki, Europhys. Lett. **32**, 355 (1995).
 - [6] M. J. Graf, S.-K. Yip, J. A. Sauls, and D. Rainer, Phys. Rev. B **53**, 15147 (1996).
 - [7] L. Taillefer, B. Lussier, R. Gagnon, K. Behnia, and H. Aubin, preprint.
 - [8] A. A. Abrikosov and V. M. Genkin, Sov. Phys.-JETP **38**, 417 (1974).
 - [9] T. P. Devereaux and D. Einzel, Phys. Rev. B **51**, 16336 (1995).
 - [10] T. P. Devereaux, D. Einzel, B. Stadlober, and R. Hackl, Phys. Rev. Lett. **72**, 3291 (1994).
 - [11] W. C. Wu (unpublished).
 - [12] M. C. Krantz and M. Cardona, Phys. Rev. Lett. **72**, 3290 (1994).
 - [13] T. P. Devereaux and A. P. Kampf, Int. J. Mod. Phys. B, to appear.
 - [14] T. P. Devereaux, D. Einzel, B. Stadlober, R. Hackl, D. H. Leach, and J. J. Neumeier, Phys. Rev. Lett. **72**, 396 (1994).
 - [15] D. Branch, PhD. Thesis, McMaster University, 1997 (unpublished).